College Algebra Quick Reference Sheet

Set Notation

Interval Notation	Set-Builder Notation
(a, b)	$\{ x \mid a < x < b \}$
$[a, b]$	$\{ x \mid a \le x \le b \}$
$[a, b)$	$\{ x \mid a \le x < b \}$
$(a, b]$	$\{ x \mid a < x \le b \}$
(a, ∞)	$\{ x \mid a < x \}$
$[a, \infty)$	$\{ x \mid a \le x \}$
$(-\infty, b)$	$\{ x \mid x < b \}$
$(-\infty, b]$	$\{ x \mid x \le b \}$

Set Operations

Operation	Elements	Logic
Union ⋃	All	OR
Intersection ⋂	Common	AND

Coordinate Plane Quadrants

II	I
III	IV

Distance and Midpoint Formulas

If $P_1=(x_1,y_1)$ and $P_2=(x_2,y_2)$ are two points, the distance between them is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the midpoint coordinates are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Intercepts of an Equation

x-intercepts	Set $y = 0$; solve for x
y-intercepts	Set $x = 0$; solve for y

Symmetry of the Graph of an Equation

Type	Mathematical	Geometrical
x-axis	Unchanged when y replaced by $-y$	Unchanged when reflected about x-axis
y-axis	Unchanged when x replaced by $-x$	Unchanged when reflected about y-axis
origin	Unchanged when y replaced by $-y$ & x replaced by $-x$	Unchanged when rotated 180° about origin

Function Notation $y = f(x)$

Domain	Set of all valid x
Range	Set of all valid y

Function Arithmetic

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Transformations of Graphs of Functions

$$\boldsymbol{y = g(x) = af(bx + h) + k}$$

	Horizontal	Vertical
Shift	$h > 0$ (left) $h < 0$ (right)	$k > 0$ (up) $k < 0$ (down)
Reflect	$b < 0$ (y-axis)	$a < 0$ (x-axis)
Scale	$\lvert b \rvert > 1$ (compress)	$\lvert a \rvert > 1$ (expand)

1. **Subtract h** from each of the **x-coordinates** of the points on the graph of f. This results in a horizontal shift to the **left** if h > 0 (positive h) or **right** if h < 0 (negative h).

2. **Divide** the **x-coordinates** of the points on the graph obtained in **Step 1** by **b**. This results in a horizontal scaling, but may also include a reflection about the y-axis if b < 0 (negative b).

3. **Multiply** the **y-coordinates** of the points on the graph obtained in **Step 2** by ***a***. This results in a vertical scaling, but may also include a reflection about the x-axis if $a < 0$ (negative a).

4. **Add k** to each of the **y-coordinates** of the points on the graph obtained in **Step 3**. This results in a vertical shift **up** if k > 0 (positive k) or **down** if k < 0 (negative k).

Properties of Equality

If $a = b$ then $a + c = a + c$ and $a - c = a - c$

If $a = b$ and $c \neq 0$ then $ac = bc$ and $\frac{a}{c} = \frac{b}{c}$

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Lines or Linear Functions

Slope of Line through points (x_1, y_1) & (x_2, y_2)

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-Intercept Form - slope m and point $(0, b)$

$$y = f(x) = mx + b$$

Point-Slope Form - slope m and point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

or

$$y = f(x) = m(x - x_1) + y_1$$

Horizontal Line through point $(0, b)$

$$y = f(x) = b$$

Vertical Line through point $(a, 0)$

$$x = a$$

Average Rate of Change

The average rate of change m for function $y=f(x)$ between $x=a$ and $x=b$ is

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Absolute Value Properties

$$|a| = \begin{cases} -a, & \text{if } a < 0 \\ a, & \text{if } a \ge 0 \end{cases}$$

$\lvert a \rvert \ge 0$	$\lvert -a \rvert = \lvert a \rvert$
$\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	$\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert}$

Absolute Value Function as a Piecewise-Defined Function

$$f(x) = |g(x)| \rightarrow f(x) = \begin{cases} -g(x), & g(x) < 0 \\ g(x), & g(x) \ge 0 \end{cases}$$

Absolute Value Equations and Inequalities

If c is a positive number:

$|x| = c \rightarrow x = -c$ or $x = c$

$|x| < c \rightarrow -c < x < c$

$|x| \le c \rightarrow -c \le x \le c$

$|x| > c \rightarrow x < -c$ or $x > c$

$|x| \ge c \rightarrow x \le -c$ or $x \ge c$

Parabolas or Quadratic Functions

General Form $\quad y = f(x) = ax^2 + bx + c$

The graph has a smile if a is positive and a frown if a is negative, and has a vertex at coordinates:

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

Vertex Form $\quad y = f(x) = a(x - h)^2 + k$

The graph has a smile if a is positive and a frown if a is negative, and has a vertex at (h, k).

Special Factoring Formulas

$x^2 + a^2 =$ No Real Factors

$x^2 - a^2 = (x + a)(x - a)$

$x^2 + 2ax + a^2 = (x + a)^2$

$x^2 - 2ax + a^2 = (x - a)^2$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

Special Product Formulas

$(x + a)(x - a) = x^2 - a^2$

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$

$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

Quadratic Formula

Solve $ax^2 + bx + c = 0, a \neq 0$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$, then 2 real unequal solutions

If $b^2 - 4ac = 0$, then 2 real duplicate solutions

If $b^2 - 4ac < 0$, then no real solutions

Factored Form for real factors:

$$y = f(x) = a(x - x_1)(x - x_2)$$

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End Behavior of a Polynomial Function $f(x) = ax^n + \cdots$

n	a	Behavior
odd	$a > 0$	$x \to -\infty, f(x) \to -\infty$ $x \to \infty, f(x) \to \infty$
odd	$a < 0$	$x \to -\infty, f(x) \to \infty$ $x \to \infty, f(x) \to -\infty$
even	$a > 0$	$x \to -\infty, f(x) \to \infty$ $x \to \infty, f(x) \to \infty$
even	$a < 0$	$x \to -\infty, f(x) \to -\infty$ $x \to \infty, f(x) \to -\infty$

Multiplicities of Real Zeros of a Polynomial Function $f(x) = (x-a)^m$

m	Behavior
odd	Crosses the *x*-axis
even	Touches the *x*-axis

Rational Functions

Vertical Asymptotes (No Holes)

If a factor (*x*-*a*) appears in the denominator (but not in the numerator), the line *x*=*a* is a vertical asymptote.

Horizontal Asymptote

If the degree of the numerator is less than the degree of the denominator, then there is a horizontal asymptote at $y = 0$ (*x*-axis).

If the degree of the numerator is the same as the degree of the denominator, then there is a horizontal asymptote at *y*= (leading coefficient of numerator) / (leading coefficient of denominator).

If the degree of the numerator is greater than the degree of the denominator, then there is not a horizontal asymptote.

Composition of Functions

$$(f \circ g)(x) = f(g(x))$$

Inverse Function

Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain B and range A. Each point with coordinates (a, b) in f has a corresponding point (b, a) in f^{-1}.

Steps for Finding the Inverse Function

1. Replace $f(x)$ with *y*.
2. Interchange *x* and *y*.
3. Solve for *y*.
4. Replace *y* with $f^{-1}(x)$.

Inverse Function Property

Let f be a one-to-one function with domain A and range B. The inverse function f^{-1} satisfies the following cancelation properties.

$$f^{-1}(f(x)) = x \text{ for every } x \text{ in } A$$
$$f(f^{-1}(x)) = x \text{ for every } x \text{ in } B$$

Radical Properties

$\sqrt[n]{a} = a^{1/n}$	$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
$\sqrt[n]{a^n} = a$ if n is odd	
$\sqrt[n]{a^n} = \lvert a \rvert$ if n is even	

Exponent Laws and Properties

$a^0 = 1, a \neq 0$	$a^{-n} = \frac{1}{a^n}, a \neq 0$
$a^m a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$
$(a^m)^n = a^{mn}$	$a^{m/n} = \sqrt[n]{a^m}$
$(ab)^n = a^n b^n$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$	$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

Logarithm Definition

$\log_a x = y \leftrightarrow a^y = x$ where $a > 0$ & $a \neq 1$

Logarithm Example

$\log_2 32 = 5 \leftrightarrow 2^5 = 32$

Special Logarithms

Common Logarithm	$\log x = \log_{10} x$
Natural Logarithm	$\ln x = \log_e x$
where $e = 2.718281828459045 \ldots$	

Logarithm Properties

$\log_a 1 = 0$	$\ln 1 = 0$
$\log_a a = 1$	$\ln e = 1$
$\log_a a^x = x$	$\ln e^x = x$
$a^{\log_a x} = x$	$e^{\ln x} = x$

Laws of Logarithms

Product Rule	$\log_a(AB) = \log_a A + \log_a B$
Quotient Rule	$\log_a(A/B) = \log_a A - \log_a B$
Power Rule	$\log_a(A^C) = C \log_a A$

Logarithm Change of Base Formula

$$\log_a x = \frac{\log x}{\log a} \text{ or } \log_a x = \frac{\ln x}{\ln a}$$

Steps to Solve an Exponential Equation

1. Isolate the exponential function.
2. Take the appropriate logarithm of both sides.
3. Use the inverse function property.
4. Solve for the variable.

Steps to Solve a Logarithmic Equation

1. Isolate the logarithmic function.
2. Use the appropriate base to raise both sides.
3. Use the inverse function property.
4. Solve for the variable.
5. Remove false answers (look for domain errors).

Arithmetic Sequence

Definition: $a, a+d, a+2d, a+3d, a+4d, \ldots$

n[th] **term:** $a_n = a + (n-1)d$

n[th] **partial sum:**

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ or } S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Geometric Sequence

Definition: $a, ar, ar^2, ar^3, ar^4, \ldots$

n[th] **term:** $a_n = ar^{n-1}$

n[th] **partial sum:**

$$S_n = a\frac{1-r^n}{1-r} \text{ or } S_n = \frac{a_1 - a_{n+1}}{1-r}, r \neq 1$$

Finance Formulas

For all formulas:

- A_f is the future amount
- A_p is the present amount
- t is the number of years
- r is the annual interest rate (decimal)
- n is the number of periods in a year
- $i = r/n$ is the interest rate per period
- R is the periodic payment amount

Simple Interest

$$A_f = A_p(1 + rt)$$

Compound Interest

$$A_f = A_p(1+i)^{nt} \text{ or } A_f = A_p\left(1 + \frac{r}{n}\right)^{nt}$$

Continuously Compounded Interest

$$A_f = A_p e^{rt}$$

Future Value of an Annuity

$$A_f = R\frac{(1+i)^{nt} - 1}{i}$$

Present Value of an Annuity

$$A_p = R\frac{1 - (1+i)^{-nt}}{i}$$

Payment Amount of a Loan

$$R = A_p\frac{i}{1 - (1+i)^{-nt}}$$